**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Since work beings 10 mins after the car is dropped, the time left to complete work is 50 mins.Probability that service manager cannot meet his commitment=P(X>50)=1-Pr(x<=50)(X is the time taken to complete work).Convert 50 to z-score.

Standard normal variable Z=(X- *μ)/σ=(x-45)/8*

*P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)=0.73237(the number in z-table)*

*Probability that service manager will not meet his commitment is: 100-73.237=26.763%=0.2676*

*S0,the answer is B.*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Let's analyze the given statements:

1. More employees at the processing center are older than 44 than between 38 and 44.

Given, mean *μ* = 38 and Standard deviation *σ=6.*

1. Probability of employees>44=Pr(x>44)=1-Pr(x<=44)

Z=(X-*μ)/* *σ=(x-38)/6*

Pr(x<=44)=Pr(z<=(44-38)/6)=Pr(Z<=1)=0.84134=84.134%

Probability that employees will be greater than 44=100-84.134=15.866

Probability of employees between 38 & 44=Pr(x<=44)-Pr(x>=38)

Here,Pr(x<=44)=0.84134

Pr (x>=38)=Pr(z>=(38-38)/6)=Pr(z>=0)=0.5

Therefore,Pr(x<=44)-Pr(x>=38)=0.84134-0.5=0.34134=34.134%

So, the statement “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. For this statement, we need to find the proportion of employees under the age of 30.

Z-score for 30: *Z*=30−38​/6=-8​/6=−1.33

Using the standard normal distribution table, the area to the left of Z = -1.33 is about 0.0912 (or approximately 9.12%).

Therefore, if 9.12% of 400 employees are under the age of 30, we can calculate the expected number:

Expected number = Proportion under 30 × Total number of employees Expected number = 0.0912 × 400 ≈ 36.48

So, the statement is **True**. The expected number of employees under the age of 30 at the center is approximately 36.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2. µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The probability of getting value between a & b is 0.99

So,the probability of getting value outside a & b is 1-0.99=0.01

The probability towards left of a=-0.01/2=-0.05

The probability towards right of b=0.01/2=0.05

Since we have probabilities of a & b,we need calculate the probability of X-the

random variable at a & bwhich has these probabilities.

By finding Standard Normal Variable(z),need to calculate X:

Z=(X=Mue)/Sigma

For a probability 0f 0.005,z values is -2.57

Z\**σ+μ=x*

-(2.57)\*20+100=151.4

(-2.57)\*20+100=48.6

Option D is the correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for

the annual profit of the company.

Ans: Mean profit is Rs 540 Million

Standard Deviation is Rs 225 Million

Range is Rs State norms interval (0.95,540,225)

Range is Rs 99.0081034,980.991896

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: Formula X=X=*μ+Zσ; wherein from z table,5 percentile=-1.645*

*X=540(-1.645)\*225*

*X=169.875*

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of division 1 making a loss p(X<0)

Stats.norm.cdf(0,5,3)

0.0477903

Probability of division 2 making a loss p(X<0)

Stats.norm.cdf(0,7,4)

0.0400591